

Hydromagnetic Kelvin-Helmholtz Instability in the Presence of Suspended Particles and Finite Larmor Radius Effect

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A linear analysis of the combined influence of a finite ion Larmor radius and suspended particles on Kelvin-Helmholtz instability in the presence of a uniform magnetic field is carried out. The magnetic field is assumed to be uniform and transverse to the direction of streaming. The medium is assumed to be incompressible. Certain simplifying assumptions are made for the motion of the suspended particles. A dispersion relation for such a medium has been obtained using appropriate boundary conditions. The stabilizing effect of a finite Larmor radius has been reasserted in the absence of the suspended particles. A stability criterion for the medium is derived, which is found to be independent of the presence of the suspended particles. Similarly a condition of instability of the system is also derived. Numerical analysis is presented in a few limiting cases of interest. Furthermore, growth rates of unstable modes of the configuration with increasing relaxation frequency of the particles and finite Larmor radius have been evaluated analytically. It is shown that the finite Larmor radius in the presence of the suspended particles destabilizes a certain wave number band which is stable otherwise. Implications of the suspended particles on the growth rate of unstable modes are discussed in the limit of vanishing ion Larmor radius.

Key words: Plasma instability, Magnetohydrodynamic instability, Suspended particles, Kelvin-Helmholtz instability, Finite Larmor radius effect.

I. Introduction

Kelvin-Helmholtz (KH) instability arises due to velocity shear in the fluid. Chandrasekhar [1] has made a comprehensive survey of the hydromagnetic version of this instability. An excellent review of the KH instability has been presented by Gerwin [2]. The KH instability is of interest in investigating a variety of space, astrophysical and geophysical situations involving sheared plasma flows. The stability of the magnetopause, which is a surface of tangential discontinuity between solar wind and the magnetosphere, has been discussed by Sen [3]. The 'edge oscillation' in a Q-machine plasma has been identified to be a transverse KH instability, and suppression of this instability has been observed experimentally by Sugai and Sato [4].

In the investigation of the hydromagnetic KH instability problem, many authors used the approximation of zero Larmor radius for the charged particles, which is not realistic in a number of physical situations. Rosenbluth et al. [5] have demonstrated the stabilizing effect of the finite ion Larmor radius (FLR) on the gravitational instability of a magnetic plasma using

the Vlasov equations. Roberts and Taylor [6] have derived the FLR stabilization by making appropriate modifications in the magnetohydrodynamic equations. Recently Nagano [7] has investigated the influence of FLR on the KH instability of the magnetopause and discussed the results in comparison with experimental observations. The FLR effect on the instability of incompressible infinitely conducting superposed fluids has been considered by Kalra [8]. He has shown that the FLR stabilizes perturbations for all wave numbers. Coppi [9] has observed that the Larmor radius slows down resistive modes in a collisional plasma. Kent [10] has made a study of the stability of parallel flows in the presence of a variable horizontal magnetic field. Bhatia [11] has examined the case of two superposed fluids of variable viscosity immersed in a uniform magnetic field and found that the growth rate is suppressed on account of the viscosity. The problem of KH instability of a compressible as well as incompressible plasma has been studied by Nagano [12] to include FLR effects. The simultaneous effects of the collisional frequency and Larmor radius on hydromagnetic KH configurations are investigated by Hans [13]. He has shown that the Larmor radius has a stabilizing influence. Also, the collisional frequency has a stabilizing effect when small and a destabilizing influence when it exceeds a certain value.

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In fluid dynamics for the discussion of flow and stability problems, the effect of the suspended particles is widely considered. Saffman [14] has studied in detail a dusty gas in magnetohydrodynamics. Scanlon and Segel [15] have made a thorough study of the implication of suspended particles in hydromagnetics in the context of the Benard convection problem. Recently, Sanghvi and Chhajlani [16] have incorporated the finite resistivity effect on the Rayleigh-Taylor (RT) configuration of a stratified plasma in the presence of suspended particles and found that the particles have a stabilizing as well as a destabilizing influence under certain conditions.

Apart from these, the KH instability has been applied to the understanding of a variety of physical situations such as the physics of the solar atmosphere [17], the stripping of gas from the clusters of moving galaxies [18] and the structure of the tail of comets [19]. In addition, Choudhury and Patel [20] have made a linear stability analysis of an anisotropic sheared layer with a parallel uniform magnetic field for such problems.

Chandrasekhar [1] has treated the problem of KH instability, considering a uniform magnetic field along and transverse to the direction of streaming. He has noted that these two directions are profoundly different with respect to the development of KH instability. The magnetic field in the direction of streaming has a stabilizing influence on the KH instability whereas it does not affect the stability in the transverse direction. In this regard, therefore, it is of interest to investigate the FLR effect (which manifests itself in the form of magnetic viscosity in the fluid equations) in the presence of a uniform magnetic field along the direction of streaming or transverse to it. In the present study we have incorporated FLR corrections in the KH configuration of two superposed streaming fluids acted upon by a uniform magnetic field transverse to the direction of streaming in the presence of suspended particles and discussed various interesting implications of the FLR corrections and suspended particles.

II. Basic Equations and Linearization

We consider a model of two semi infinite homogeneous fluids separated by a plane interface (of negligible thickness) at $z = 0$, each of these regions ($z < 0$ and $z > 0$), denoted by the subscripts 1 and 2, are permeated by suspended particles of the same density, i.e. we

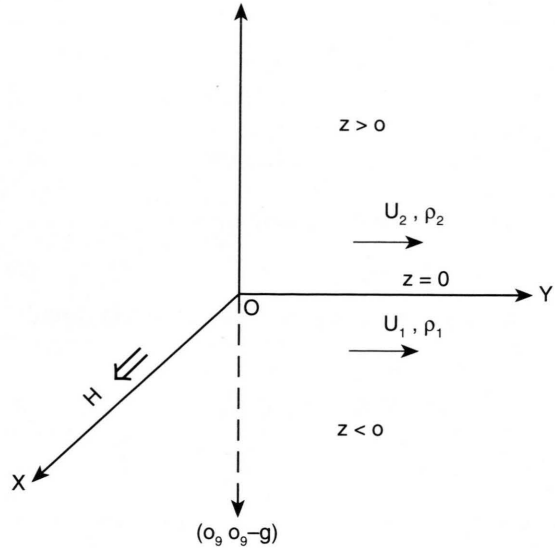


Fig. 1. Schematic diagram of the flow.

assume a homogeneous distribution of the suspended particles in both the regions (see Figure 1). Thus the medium can be regarded as a uniform mixture of gas and suspended particles. The gas is considered infinitely conducting and incompressible. The particles are assumed to be non-conducting. Let the mixture of the hydromagnetic fluid and the suspended particles stream together with a velocity $\mathbf{U}(0, U, 0)$ in a transverse magnetic field $\mathbf{H} = (H, 0, 0)$ which is essentially uniform and is acted upon by a downward gravitational field $\mathbf{g}(0, 0, -g)$.

In order to bring out essential features of the problem, we shall make certain simplifying assumptions about the motion of the suspended particles. We assume that the particles are uniform in size and spherical in shape. Let \mathbf{v} and N denote the velocity and the number density of the particles. It is supposed that the bulk concentration of the particles is very small, to that the net effect of the particles on the gas is equivalent to an extra body force $K N(\mathbf{v} - \mathbf{U})$, where \mathbf{U} is the velocity of the gas, K is a constant given by $K = 6\pi a \mu$ (Stokes drag formula), a being the particle-radius, and μ is the viscosity of the clean gas.

Thus the relevant equations of motion and continuity of the hydromagnetic fluid are

$$\rho \left[\frac{\partial \mathbf{U}}{\partial t} + (\mathbf{U} \cdot \nabla) \mathbf{U} \right] = -\nabla p - \nabla \cdot \mathbf{P} + K N(\mathbf{v} - \mathbf{U}) \quad (1)$$

$$+ \frac{1}{4\pi} (\nabla \times \mathbf{H}) \times \mathbf{H} + \rho \mathbf{g} + \mu \nabla^2 \mathbf{U},$$

$$\frac{\partial \varrho}{\partial t} + \nabla \cdot (\varrho \mathbf{U}) = 0, \quad (2)$$

where ϱ and p denote the density and the pressure of the gas, respectively. The FLR correction has been incorporated through the stress tensor \mathbf{P} in the equation of motion.

The incompressibility condition is

$$\nabla \cdot \mathbf{U} = 0. \quad (3)$$

Finally, the Maxwell's equations for a perfect conductor are

$$\frac{\partial \mathbf{H}}{\partial t} = (\mathbf{H} \cdot \nabla) \mathbf{U} - (\mathbf{U} \cdot \nabla) \mathbf{H}, \quad (4)$$

$$\nabla \cdot \mathbf{H} = 0. \quad (5)$$

The force exerted by the gas on the particles is equal and opposite to the force exerted by the particles on the gas. The buoyancy force on the particles is neglected, as its stabilizing effect is extremely small. Inter-particle distances are assumed to be very large as compared to the diameter of the particles, and so the inter-particle reactions can be ignored. The electrical and magnetic forces on the particles are also ignored. Under these restrictions the equations of motion and continuity for the particles are

$$mN \left[\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right] = KN(\mathbf{U} - \mathbf{v}), \quad (6)$$

where mN is the mass of the particles per unit volume, and

$$\frac{\partial N}{\partial t} + \nabla \cdot (N \mathbf{v}) = 0. \quad (7)$$

To investigate the stability of the configuration, we assume the following perturbations in various physical quantities:

$$\begin{aligned} \mathbf{U} &= \mathbf{U}_0 + \mathbf{u}', \quad \mathbf{v} = \mathbf{U}_0 + \mathbf{v}', \\ N &= N_0 + N', \quad \mathbf{H} = \mathbf{H}_0 + \mathbf{h}, \quad N_0 = \text{const}, \\ p &= p_0 + \delta p \quad \text{and} \quad \varrho = \varrho_0 + \delta \varrho, \end{aligned} \quad (8)$$

where the quantities with the subscript 0 denote equilibrium values and the quantities $\mathbf{u}'(u, v, w)$, \mathbf{v}' , N , $\mathbf{h}(h_x, h_y, h_z)$, δp and $\delta \varrho$ denote the perturbations in the gas velocity, the velocity of the particles, number density of the particles, magnetic field, fluid pressure p and density of the gas, respectively.

Substituting (8) into (1)–(7) and neglecting the second and higher order terms with respect to small fluc-

tuations, we obtain the following set of linearized equations for the considered composite fluids after dropping the subscript '0' from the equilibrium quantities:

$$\varrho \left[\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{U} \cdot \nabla) \mathbf{u} \right] = -\nabla \delta p - \nabla \cdot \mathbf{P} + \frac{1}{4\pi} (\nabla \times \mathbf{h}) \times \mathbf{H} + KN(\mathbf{v} - \mathbf{u}) + \mathbf{g} \delta \varrho, \quad (9)$$

$$\left[\tau \left(\frac{\partial}{\partial t} + \mathbf{U} \cdot \nabla \right) + 1 \right] \mathbf{v} = \mathbf{u}, \quad (10)$$

$$\frac{\partial}{\partial t} \delta \varrho + (\mathbf{U} \cdot \nabla) \delta \varrho + (\mathbf{u} \cdot \nabla) \varrho = 0, \quad (11)$$

$$\frac{\partial \mathbf{h}}{\partial t} + (\mathbf{U} \cdot \nabla) \mathbf{h} = (\mathbf{H} \cdot \nabla) \mathbf{u}, \quad (12)$$

$$\nabla \cdot \mathbf{u} = 0, \quad \nabla \cdot \mathbf{h} = 0, \quad (13)$$

where $\tau = m/K$ denotes the relaxation time for the suspended particles. In writing (9) we have assumed that the viscous term $\mu \nabla^2 \mathbf{u}$ is negligibly small as compared to the viscous drag force $KN(\mathbf{v} - \mathbf{u})$. It must be mentioned here that we have dropped primes from the perturbation quantities \mathbf{u}' , \mathbf{v}' , and N' which have been defined in (8). The stress tensor \mathbf{P} has the following components for the horizontal magnetic field $\mathbf{H}(H, 0, 0)$ (cf. Roberts and Taylor [6]):

$$\begin{aligned} P_{xx} &= 0, \\ P_{yy} &= -\varrho v_0 \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \right), \\ P_{zz} &= \varrho v_0 \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \right), \\ P_{xy} &= P_{yx} = -2\varrho v_0 \left(\frac{\partial u}{\partial z} \right), \\ P_{xz} &= P_{zx} = 2\varrho v_0 \left(\frac{\partial u}{\partial y} \right), \\ P_{zy} &= P_{yz} = \varrho v_0 \left(\frac{\partial v}{\partial y} - \frac{\partial w}{\partial z} \right). \end{aligned} \quad (14)$$

The parameter v_0 has the dimension but not the exact physical significance of a kinematic viscosity and is defined by $v_0 = R_L^2 \Omega_L / 4$, where R_L is the ion Larmor radius and Ω_L is the ion gyrofrequency. Let us look for solutions of the form

$$\exp(iky + nt), \quad (15)$$

where k is the wave number of perturbation along y -axis and n the growth rate of the perturbation.

On eliminating v from (9) with the help of (10) and then employing (14) and (15) on (9)–(13), we obtain the following set of equations:

$$\left[(n + ikU) + \frac{(n + ikU) \alpha_0}{[1 + \tau(n + ikU)]} \right] \varrho v \quad (16)$$

$$= -ik\delta p + 2v_0(D\varrho)(Dw) + \varrho v_0(D^2 - k^2)w,$$

$$\left[(n + ikU) + \frac{(n + ikU) \alpha_0}{[1 + \tau(n + ikU)]} \right] \varrho w$$

$$= -D\delta p - v_0(D\varrho)(Dv) - \varrho v_0(D^2 - k^2)v - ikv_0(D\varrho)w - g\delta\varrho, \quad (17)$$

$$(n + ikU)\delta\varrho = -wD\varrho, \quad (18)$$

$$ikv + Dw = 0, \quad (19)$$

$$ikh_y + Dh_z = 0, \quad (20)$$

where $\alpha_0 = mN/\varrho$ denotes the mass concentration of the particles and $D = d/dz$.

Equations (16) and (17) can be solved using (18)–(20) to obtain the following differential equation governing the perturbed velocity component w :

$$\begin{aligned} & [D(\varrho Dw) - k^2 \varrho w] [(n + ikU) \\ & + \alpha_0(n + ikU)/\{\tau(n + ikU + 1)\}] \\ & + 2v_0 ik [D\varrho(D^2 - k^2)w] + \frac{gk^2(D\varrho)w}{(n + ikU)} = 0. \end{aligned} \quad (21)$$

It should be remarked here that the density of the suspended particles in the two regions $z < 0$ and $z > 0$ (denoted by the subscripts 1 and 2) is assumed to be the same.

III. Dispersion Relation

We consider the case of two superposed fluids of densities $\varrho_1(z < 0)$ and $\varrho_2(z > 0)$, separated by a horizontal boundary at $z = 0$. Let the velocities of streaming of the two fluids be $U_1(0, U_1, 0)$ and $U_2(0, U_2, 0)$. Thus in the region of constant ϱ the differential equation (21) becomes

$$(D^2 - k^2)w = 0. \quad (22)$$

The solutions of (22), which remain bounded at $z = \pm \infty$ in the regions $z \leq 0$ are respectively written as

$$w_1 = A(n + ikU_1) \exp(kz), \quad (23)$$

$$w_2 = A(n + ikU_2) \exp(-kz), \quad (24)$$

where $A = \text{constant}$. To connect these solutions at $z = 0$ we require appropriate boundary conditions. Following Chandrasekhar [1], the boundary conditions across the interface are:

(i) The normal component of the velocity is continuous, which means

$$w_1/(n + ikU_1) = w_2/(n + ikU_2), \quad (25)$$

(ii) The normal component of the magnetic field is continuous. This reduces to condition (i).

(iii) The total pressure should be continuous. This condition can be obtained by integrating (21) across the physical interface $z = 0$. By the choice of solutions expressed in (23) and (24), the condition (i) as well as conditions at infinity have been met. To satisfy the boundary condition (iii) we integrate (21) across the interface $z = 0$ to obtain

$$\begin{aligned} & \Delta_0 [\varrho Dw \{(n + ikU) + \alpha_0(n + ikU)/[\tau(n + ikU) + 1]\} \\ & - 2v_0 ik^3 \Delta_0 [\varrho(n + ikU)](w/(n + ikU))_{z=0} \\ & + gk^2 \Delta_0 \varrho \left(\frac{w}{n + ikU} \right)_{z=0} = 0, \end{aligned} \quad (26)$$

where Δ_0 denotes the jump a quantity experiences at $z = 0$, and $(w/(n + ikU))_{z=0}$ refers to the unique value at $z = 0$. On substituting the solutions (23) and (24) in (26), we obtain the following dispersion relation for the composite gas particle streaming medium:

$$\begin{aligned} & n^2 + 2ink(\beta_1 U_1 + \beta_2 U_2) - gk(\beta_2 - \beta_1) \\ & + 2ikv_0 k^2(\beta_2 - \beta_1)n - k^2(\beta_1 U_1^2 + \beta_2 U_2^2) \\ & + 2v_0 k^3(\beta_1 U_1 - \beta_2 U_2) \\ & + \frac{\beta_1 \alpha_1 (n + ikU_1)^2}{[1 + \tau(n + ikU_1)]} + \frac{\beta_2 \alpha_2 (n + ikU_2)^2}{[1 + \tau(n + ikU_2)]} = 0, \end{aligned} \quad (27)$$

where we have written

$$\begin{aligned} \alpha_1 &= mN/\varrho_1, & \alpha_2 &= mN/\varrho_2 \\ \beta_1 &= \varrho_1/(\varrho_1 + \varrho_2), & \beta_2 &= \varrho_2/(\varrho_1 + \varrho_2). \end{aligned}$$

The above equation represents the combined influence of the suspended particles and the finiteness of ion Larmor radius on the hydromagnetic KH instability of two superposed fluids. In the absence of the suspended particles ($\tau = 0, \alpha_1 = \alpha_2 = 0$), this reduces to the dispersion relation obtained by Singh and Hans [21]. Note that the effect of the suspended particles enters into the dispersion relation (27) through two parameters α_0 and τ measuring the mass concentration and the relaxation time of the particles.

IV. Discussion

A. General Configuration

In this subsection we treat the configuration in the absence of suspended particles and FLR effects. The dispersion equation (27) for this case, on substituting ($v_0 = 0$, $\alpha_1 = \alpha_2 = 0$), reduces to

$$n^2 + 2ink(\beta_1 U_1 + \beta_2 U_2) + gk(\beta_1 - \beta_2) - k^2(\beta_1 U_1^2 + \beta_2 U_2^2) = 0. \quad (28)$$

The roots of (28) are given by

$$n = ik(\beta_1 U_1 + \beta_2 U_2) \pm \sqrt{k^2 \beta_1 \beta_2 (U_1 - U_2)^2 - gk(\beta_1 - \beta_2)}. \quad (29)$$

(a) $\beta_1 > \beta_2$ (Stable RT case):

We find from (29) that the KH instability is suppressed if

$$k^2 \beta_1 \beta_2 (U_1 - U_2)^2 < gk(\beta_1 - \beta_2), \quad (30)$$

since, under the above restriction (29) will not allow any real positive root of n which implies stability of the system. Thus we conclude that the considered KH configuration is stabilized for the wave numbers determined by the inequality (30). Also we find that instability results for all wave numbers satisfying the condition

$$k^2 \beta_1 \beta_2 (U_1 - U_2)^2 > gk(\beta_1 - \beta_2), \quad (31)$$

From (31), the critical wave number for an instability is

$$k_c = \frac{g(\beta_1 - \beta_2)}{\beta_1 \beta_2 (U_1 - U_2)^2}, \quad (32)$$

which is identical to Chandrasekhar [1].

(b) $\beta_1 < \beta_2$ (Unstable RT case):

In this case it is easy to see from (29) that the KH configuration remains always unstable as one of the roots of (28) is complex with positive real part. It is clear from the above analysis that interchange perturbations (KH) remain unaffected by the presence of a magnetic field.

It is elucidating to consider the case of two streaming fluids in the absence of g (gravitational force), for which we have

$$n = -ik(\beta_1 U_1 + \beta_2 U_2) \pm \sqrt{k^2 \beta_1 \beta_2 (U_1 - U_2)^2}, \quad (33)$$

which means the system in the absence of gravity is always unstable, irrespective of the magnitude and direction of streaming velocities.

Corresponding to the case of static fluids ($U_1 = U_2 = 0$) under gravity (RT configuration), one obtains from (28)

$$n^2 = gk(\beta_2 - \beta_1). \quad (34)$$

The system is stable or unstable according to $\beta_1 > \beta_2$ or $\beta_1 < \beta_2$.

B. Static Configurations

In the present subsection we deal with the case of nonstreaming superposed hydromagnetic fluids of different densities including suspended particles. In order to discuss implications of the presence of particles, we analyze the case for vanishing FLR ($v_0 = 0$). In this case the dispersion relation (27) can be written as

$$\tau n^3 + n^2 [1 + \alpha_1 \beta_1 + \alpha_2 \beta_2] - ngk(\beta_2 - \beta_1)\tau - gk(\beta_2 - \beta_1) = 0. \quad (35)$$

Introducing the relaxation frequency parameter $f_s (= 1/\tau)$ of the suspended particles and simplifying the above equation, we obtain

$$n^3 + n^2 f_s (1 + 2\alpha') - ngk(\beta_2 - \beta_1) - gk f_s (\beta_2 - \beta_1) = 0, \quad (36)$$

where $\alpha' = mN/\rho_1 + \rho_2$.

We can distinguish two cases: (i) clean configuration stable ($\beta_1 > \beta_2$) – we find that (36) does not admit any real positive or complex root with real positive part implying stability (necessary condition of Hurwitz's criterion). Thus the stable configuration remains stable even in the presence of suspended particles.

(ii) Clean configuration unstable ($\beta_1 < \beta_2$) – when the upper fluid is heavier than the lower one, then (36) will necessarily possess one real positive root (n_0) which leads to an instability of the system.

We obtain dn_0/df_s (growth rate with increasing relaxation frequency of the suspended particles) from (36)

$$\frac{dn_0}{df_s} = \frac{-[(1 + 2\alpha')n_0^2 - gk(\beta_2 - \beta_1)]}{[3n_0^2 + 2n_0 f_s (1 + 2\alpha') - gk(\beta_2 - \beta_1)f_s]}. \quad (37)$$

The growth rate turns out to be negative if

$$(1 + 2\alpha')n_0^2 > gk(\beta_2 - \beta_1) \quad (38)$$

and

$$3n_0^2 > gk(\beta_2 - \beta_1) \quad (39a)$$

hold simultaneously. We can therefore, conclude that the growth rate of unstable RT modes is decreased with increasing relaxation frequency of the particles. This means under the restrictions (38), (39a) the particles have a stabilizing influence on the configuration.

The dispersion relation (36) can be written in dimensionless form by the substitutions

$$\hat{n} = \frac{n}{\sqrt{gk}}, \quad \hat{f}_s = \frac{f_s}{\sqrt{gk}}.$$

Thus we get

$$(39b)$$

$$\hat{n}^3 + \hat{n}^2 \hat{f}_s (1 + 2\alpha') - \hat{n}(\beta_2 - \beta_1) - \hat{f}_s(\beta_2 - \beta_1) = 0,$$

where

$$\alpha' = KN\tau/\varrho_1 + \varrho_2.$$

As an illustration of the implications of suspended particles on the stability of the RT mode we have numerically solved (39b) for positive real roots for various values of the non-dimensional parameters \hat{f}_s and α' (which characterize the influence of suspended particles). Figure 2 clearly shows that the growth rate n (positive real root of n^2) decreases with increasing relaxation frequency of the suspended particles (f_s) as well as with increasing density (α') of the particles. We have kept $(\beta_2 - \beta_1) = 0.6$. It may be remarked here

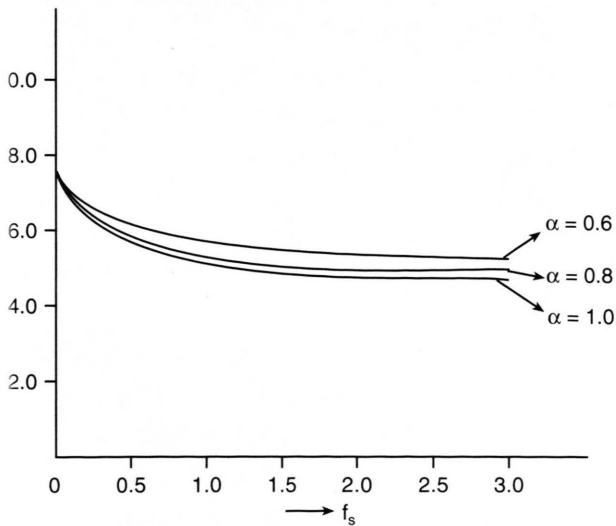


Fig. 2. The growth rate (\hat{n}) plotted against relaxation frequency for $\alpha = 0.6, 0.8$, and 1.0 .

that an increase of $f_s (= 6\pi\mu a/m)$ implies an increase in the size of the particles (a), as we have assumed, μ , ϱ , and m to be constants. Thus we can conclude that both increasing relaxation frequency of the particles and their density have a stabilizing influence on the considered RT configuration. In other words, as the size of the particles (of constant mass) increases, the growth rate of unstable RT modes decreases, which is in conformity with the analytical results on the influence of the particles obtained in the beginning of this subsection.

C. KH Instability Including FLR and Suspended Particles

To apprehend implications of FLR corrections on the KH instability we consider two streaming fluids in the presence of a uniform magnetic field transverse to the direction of streaming neglecting the effect of particles. We assume the streaming velocities of two fluids to be U_1 and U_2 , their densities being equal. The general dispersion relation (27) in this case becomes

$$n^2 + 2ink(U_1 + U_2) - k^2 \frac{(U_1^2 + U_2^2)}{2} + \nu_0 k^3 (U_1 - U_2) = 0. \quad (40)$$

The solution of the above dispersion relation is

$$n = -\frac{ik}{2}(U_1 + U_2) \pm \sqrt{\frac{k^2}{4}(U_1 - U_2)^2 - \nu_0 k^3 (U_1 - U_2)}. \quad (41)$$

If $U_1 > U_2$ (the lower fluid is streaming faster than the upper fluid), the medium is destabilized for wave numbers determined by the inequality

$$(U_1 - U_2)/4\nu_0 > k. \quad (42)$$

Thus the medium is stable for wave numbers $k_c < k$, where

$$k_c = (U_1 - U_2)/4\nu_0. \quad (43)$$

Hence we conclude that the FLR tends to stabilize the configuration and the critical wave number depends upon the relative velocity of the two fluids. On the contrary, if $U_1 < U_2$ the system is stable for $k < k_c < 0$. Furthermore, if we solve the dispersion equation (27) corresponding to fluids of equal densities and streaming velocities U and $-U$ in the absence

of suspended particles, we obtain the relation

$$n^2 = k^2 U^2 - 2\nu_0 k^2 (kU). \quad (44)$$

It corresponds to the KH model when there is a tangential discontinuity in velocities in a uniform plasma. This result has been obtained by Kalra [8] and also by Nagano [12]. We observe that the FLR has a stabilizing influence as it reduces the frequency of oscillation of the system. The criterion for instability is

$$k^2 U^2 > 2\nu_0 k^3 U, \quad (45)$$

which yields the critical wave number

$$k'_c = U/2\nu_0.$$

Here we note that the FLR effect stabilizes the perturbations for $k > k'_c$. On comparing this with (43), we find that the critical wave numbers are different for fluids streaming with different velocities (U_1, U_2) or with the same speeds in opposite directions ($U, -U$).

Now we solve (44) numerically for positive roots in order to illustrate the influence of FLR on the KH configuration. Note that we are analysing interchange ($k \perp H$) perturbations. To do this, we put (44) in the non-dimensional form

$$\hat{n}^2 = (\hat{k} \hat{U})^2 - 2\hat{\nu}_0 \hat{k}^3 \hat{U}, \quad (46)$$

where we have introduced the non-dimensional parameters

$$\hat{n} = \frac{nL}{V_a}, \quad \hat{k} = kL, \quad \hat{U} = U/V_a \quad \text{and} \quad \hat{\nu}_0 = \nu_0/V_a L.$$

V_a and L denote the Alfvén speed and the characteristic length, respectively. Figure 3 shows the variation of the growth rate n (positive real part of \hat{n}) as a function of the wave number k for different values of ν_0 , taking $U = 5.0$ (fixed). The broken lines correspond to the case of ideal MHD. We also notice that the growth rate first increases for small \hat{k} , attains a maximum but remains smaller than the usual MHD approximation and thereafter decreases and becomes zero for the corresponding critical wave number. The foregoing analysis indicates that interchange perturbations are stabilized by the FLR effect for $k > k'_c$, where $k'_c = U/2\nu_0$. Furthermore, it is observed that, as the FLR increases, the domain of instability is also reduced.

Finally, to study the combined influence of the FLR and suspended particles, we specialize the dispersion relation (27) when identical gas particle composite media occupy the two regions $z < 0$ and $z > 0$. The

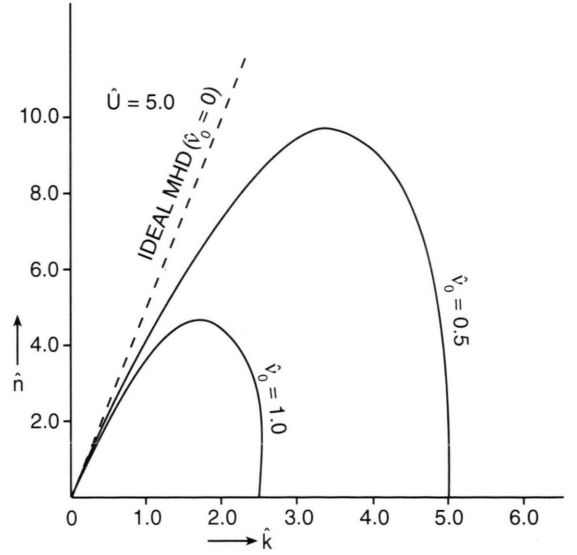


Fig. 3. The growth rate \hat{n} plotted against \hat{k} for $\nu_0 = 0, 0.5$, and 1.

streaming velocities in the two halves are assumed to be U and $-U$ in the presence of a uniform magnetic field transverse to the direction of streaming. In this case we have to put the following values in the dispersion relation (27):

$$\alpha_1 = \alpha_2 = \alpha_0, \quad \beta_1 = \beta_2 = 1/2, \\ U_1 = U, \quad U_2 = -U.$$

Thus, we obtain the following dispersion relation corresponding to the above case:

$$\tau^2 n^4 + \tau n^3 (2 + \alpha_0) + [(1 + \alpha_0) + 2\nu_0 U k^3 \tau^2] n^2 \\ + \tau n [k^2 U^2 \alpha_0 + 2(2\nu_0 U k^3 - k^2 U^2)] \\ + [(2\nu_0 U k^3 - k^2 U^2 - \alpha_0 k^2 U^2) \\ + \tau^2 k^2 U^2 (2\nu_0 U k^3 - k^2 U^2)] = 0. \quad (47)$$

The above dispersion relation, on writing $f_s = 1/\tau$ (the relaxation frequency parameter of the suspended particles), becomes

$$n^4 + f_s n^3 (2 + \alpha_0) + [f_s^2 (1 + \alpha_0) + 2\nu_0 U k^3] n^2 \\ + f_s n [k^2 U^2 \alpha_0 + 2(2\nu_0 U k^3 - k^2 U^2)] \\ + [(2\nu_0 U k^3 - k^2 U^2 - \alpha_0 k^2 U^2) f_s^2 \\ + k^2 U^2 (2\nu_0 U k^3 - k^2 U^2)] = 0. \quad (48)$$

This dispersion equation enables us to examine the transverse KH configuration in the presence of the FLR effect and suspended particles simultaneously.

We find the derivative of the growth rate of the unstable RT mode (n_0) with increasing FLR (v_0). From (48) we thus

$$\frac{dn_0}{dv_0} = \frac{-[2n_0^2 U k^3 + 4n_0 f_s U k^3 + 2U k^3 (f_s^2 + k^2 U^2)]}{[4n_0^3 + a_1 n_0^2 + a_2 n_0 + a_3]}, \quad (49)$$

where

$$\begin{aligned} a_1 &= 3f_s(2 + \alpha_0), \\ a_2 &= 2[(1 + \alpha_0)f_s^2 + 2v_0 U k^3], \\ a_3 &= [k^2 U^2 \alpha_0 + 2(2v_0 k^3 U - k^2 U^2)]f_s. \end{aligned}$$

The growth rate is negative if

$$[k^2 U^2 \alpha_0 + 2(2v_0 U k^3 - k^2 U^2)] > 0 \quad (50a)$$

and it will be positive if

$$4n_0^3 + 3f_s(2 + \alpha_0)n_0^2 + 2[(1 + \alpha_0)f_s^2 + 2v_0 U k^3]n_0 + [k^2 U^2 \alpha_0 + 2(2v_0 U k^3 - k^2 U^2)] < 0. \quad (50b)$$

Note that the condition (50a) is identical to the condition (45) when $\beta_1 > \beta_2$, under which we have shown a stabilizing influence of the suspended particles on the configuration. Thus we find that the growth rate (dn_0/dv_0) of unstable modes of the consid-

ered system is reduced with the increasing FLR (v_0) under the condition (50a), whereas it is enhanced with the increasing FLR if (50b) is satisfied. In other words, the conditions (50a), (50b) define regions where the finite ion Larmor radius has a stabilizing or destabilizing influence on the growth rate of unstable mode. It is also seen from (50a), (50b) that the conditions determining the region involve the parameters f_s and α_0 of the suspended particles and Larmor radius v_0 .

D. Implications of Suspended Particles on the Medium

In this subsection we carry out an analysis with vanishingly small FLR ($v_0 = 0$) for investigating implications of suspended particles in a more complete manner. The dispersion equation (48) in the case of zero Larmor radius ($v_0 = 0$) reduces to

$$\begin{aligned} n^4 + f_s(2 + \alpha_0)n^3 + (1 + \alpha_0)f_s^2 n^2 \\ + f_s n(k^2 U^2 \alpha_0 - 2k^2 U^2) \\ - [f_s^2 k^2 U^2(1 + \alpha_0) + k^4 U^4] = 0. \end{aligned} \quad (51)$$

Clearly the absolute term of the equation is always negative, which means it will admit at least one real positive root. However, it is of interest to evaluate the derivative of the growth rate of an unstable mode (n_0) of propagation. From (51), we find

$$\frac{dn_0}{df_s} = \frac{-[n_0^3(2 + \alpha_0) + 2n_0^2 f_s(1 + \alpha_0) - n_0 k^2 U^2(2 - \alpha_0) - 2f_s k^2 U^2(1 + \alpha_0)]}{[4n_0^3 + 3n_0^2 f_s(2 + \alpha_0) + 2n_0(1 + \alpha_0)f_s^2 - k^2 U^2 f_s(2 - \alpha_0)]}. \quad (52)$$

In writing (51) we have taken note of the fact that $\alpha_0 = \frac{mN}{\rho}$ cannot exceed 1. Let us now consider the inequalities

$$\begin{aligned} n_0^3(2 + \alpha_0) + 2n_0^2 f_s(1 + \alpha_0) \\ \geq k^2 U^2 [n_0(2 - \alpha_0) + 2f_s(1 + \alpha_0)] \end{aligned} \quad (53)$$

and

$$\begin{aligned} 4n_0^3 + 3n_0^2 f_s(2 + \alpha_0) + 2n_0(1 + \alpha_0)f_s^2 \\ \geq f_s k^2 U^2(2 - \alpha_0). \end{aligned} \quad (54)$$

If both upper signs of the inequalities (53) and (54) are satisfied simultaneously, we find that dn_0/df_s is negative, and if the upper and lower signs or vice versa hold then dn_0/df_s turns out to be positive. From the above analysis we find that in the absence of FLR the suspended particles reduce the growth rate of the considered KH system. A similar conclusion regarding

the effect of suspended particles has been derived by Chhajlani et al. [22] in the context of Rayleigh-Taylor instability of a stratified plasma in the presence of a uniform horizontal magnetic field.

E. Role of FLR

This section emphasizes the role of FLR on the KH model of two superposed hydromagnetic fluids in the absence of suspended particles. It is known that a magnetic field acting in the direction of streaming has, in general, a stabilizing effect on the KH configuration. A condition of stability of such a model considering superposed fluids of equal densities has been derived by Chandrasekhar [1], Shivamoggi [23] and many others, which is

$$V_a \geq U/2, \quad (55)$$

where V_a represents Alfvén speed. It was also pointed out by Chandrasekhar that a uniform magnetic field transverse to the direction of streaming does not contribute to the development of the KH instability. The assumption of zero Larmor radius is implicit in his analysis. We have incorporated the FLR corrections in the KH model considering a uniform magnetic field acting transverse to the direction of streaming. The criterion for stability of the medium

$$k v_0 \geq U/2. \quad (56)$$

This follows from the analysis carried out in the subsection IIIc, cf. (43). On comparing (55) and (56), we note an interesting feature in that the quantity $k v_0$ (which has the dimensions of velocity) appears in (56) instead of the Alfvén speed. This indicates that the FLR effect which arises due to the presence of the transverse magnetic field stabilizes the KH system in a similar manner as done by a magnetic field parallel to the direction of streaming. Thus we conclude that a magnetic field transverse to the direction of streaming does influence the KH instability when the FLR corrections are included in the analysis. In view of the foregoing, we assert that the FLR stabilization for interchange perturbations ($k \perp H$) is similar to the stabilization due to a magnetic field for non-interchange ($k \parallel H$) perturbations.

Thus we have made a linear analysis of the Kelvin-Helmholtz instability of two superposed infinitely conducting fluids (consisting of a uniform mixture of a gas and suspended particles) in the presence of a uniform magnetic field transverse to the direction of streaming including finite ion Larmor radius. It has been shown that the finite ion Larmor radius stabilizes the configuration by reducing the frequency of oscillation of the system in the absence of the particles. This result is in conformity with the stabilizing effect of finite Larmor radius shown by previous authors. Furthermore, a dispersion relation in the limit of vanishing FLR has been derived wherein the suspended particles are found to have a stabilizing as well as a destabilizing effect on the Kelvin-Helmholtz configuration under certain conditions. In particular, we have noted that the FLR in the case of transverse KH instability plays a similar stabilizing role as the magnetic field does for perturbations parallel to the direction of streaming cf. (55) and (56).

Acknowledgements

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